BRADGATE
Education Partnership

Teaching School Alliance

Calculation Policy

2017/18

This calculation policy has been reviewed and developed as a tool to support the teaching of calculation in KS1 and KS2 across Bradgate Education Partnership and STEP TSA. We would like to thank all those colleagues from the schools listed below that have contributed to the development of this document.

Great Dalby Primary School

Swallowdale Primary School

The Merton Primary School

The Pochin School

Introduction

This Calculation Policy has been written with the aims of the National Curriculum at the heart of it.

The National Curriculum Aims

The national curriculum for mathematics aims to ensure that all pupils:

♣ become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately

♣ **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language

♣ can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

Each of the four operations build on a solid understanding of place value, the connections between the four number operations and number sense, such as: whether they are odd or even, whether they are close to multiples of ten or if they are close together.

- Children need to use correct mathematical terminology in context and be able to verbalise their calculation strategies.

- Children need to make considered decisions as to the most appropriate methods to make mathematics more functional. They need to choose the most appropriate, fluent, efficient and accurate method to do a particular calculation.

- Children need to use concrete resources before they progress to pictorial and abstract representations. This CPA (concrete, pictorial and abstract) approach needs to be available to children throughout school, as and when necessary. Use of manipulatives (numicon, Cuisenaire, dienes, HTO counters etc.) helps reinforce understanding and provides support when calculating mentally, mentally with jottings, using expanded methods and formal written methods. Use of the bar model, number lines and part-part whole diagrams are recommended.

- Children should progress between the stages working towards formal written methods (where appropriate), once they have mastered each stage. However, they should not be hurried and, after the method has been taught, children should still be able to make their preferred choice of the most appropriate, efficient and accurate method for them. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

- As new methods of calculations are introduced, children should have the opportunity to examine them, alongside the method they have consolidated, to make connections between the methods and establish the similarities and differences between them.

This policy includes sections on: Addition, Subtraction, Multiplication and Division. It outlines progression in teaching, from mental through to formal written methods.

Addition

Models and Images

Number tracks

Bead strings

Number lines [marked and unmarked]

Base 10

Place value counters

Place value (arrow) cards

Ten frames

Numicon

Cuisenaire

Counting sticks

Hundred squares

Bar model

IT resources (e.g. MyMaths, Maths Works, ITPs)

Commutative law

Key Vocabulary

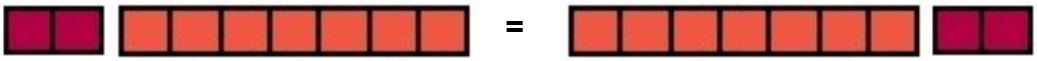
Add	Sum
Addition	Altogether
Plus	Increase
And	More
Count on	Count all
Total	

Year 1 Objectives

- **Given a number, identify one more.**
- **Read, write and interpret mathematical statements involving addition (+) and the equals (=) sign.**
- **Add one-digit and two-digit numbers within 20, including zero.**
- **Solve missing number problems (e.g. 6 + __ = 10).**

Mental Strategies

- Show how the commutative law can be used to reorder numbers when adding
  e.g. put the larger number first: **2 + 7 becomes 7+ 2**

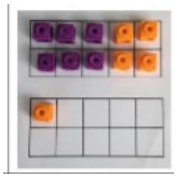


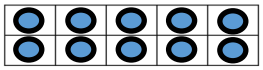
- Count on in ones or twos: **5 + 2 becomes 5 + 1 + 1 and 8 + 4 becomes 8 + 2 + 2**

Children will practise counting on from any number: 'Put five in your head and count on four.'

Initially, use a number track to count on for addition, counting on from the largest number, progress to using number lines.

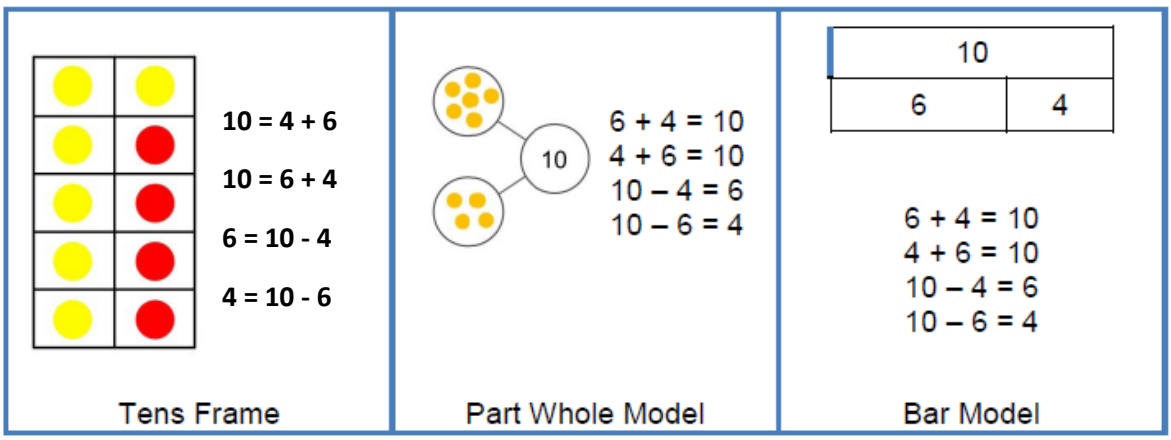
- Partition small numbers: **6 + 5 becomes 6 + 4 + 1**





- Partition and combine tens and ones: **12 + 7 becomes 10 + 2 + 7**

- Use number facts knowledge: **7 = 7 + 0 or 6 + 1 or 5 + 2, etc.**
- Number bonds to 10 and 20: **5 + 5 ; 7 + 3 ; 15 + 5 ; 16 + 4, etc.**
- Use concrete and pictorial representations to solve missing number problems:



Year 2 Objectives

Add numbers using concrete objects, pictorial representations, and mentally, including:

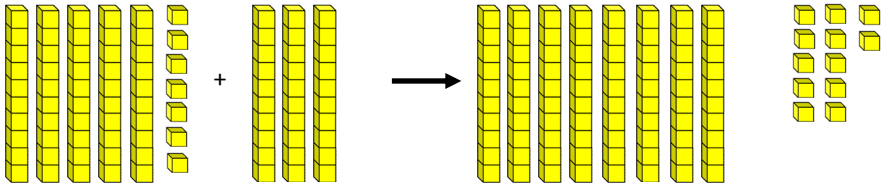
- **a two digit number and ones;**
- **a two digit number and tens;**
- **two two-digit numbers;**
- **three one-digit numbers.**

Mental Strategies

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Use place value and say 1 more and 10 more than any two digit number

- Use place value to partition: **57 + 35 as 50 + 30 = 80 then 7 + 5 =12 then 80 + 12 = 92**



- Use place value to partition then adjust: **45 + 9 becomes 45 + 10 = 55 then 55 – 1 = 54**

  45 + 21 becomes 45 + 20 = 65 then 65 + 1 = 66

- Count on in multiples of 10: **76 + 20 as 76 + 10 + 10 saying 76, 86, 96**

  e.g. using hundred squares / base ten / number lines

- Use patterns of known facts: **6 + 3 = 9, so 36 + 3 = 39 and 76 + 3 = 79**

- Bridge through 10: **57 + 5 as 57 + 3 + 2 = 62**

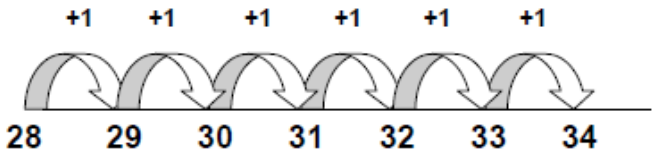
- Use number bonds to 10 knowledge when adding 3 or more single digit numbers:

  6 + 7 + 4 as 10 + 7

Written Strategies

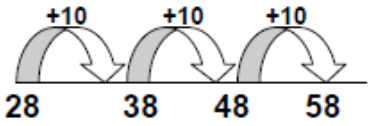
Count on in ones and tens, using an empty number line, within 100:

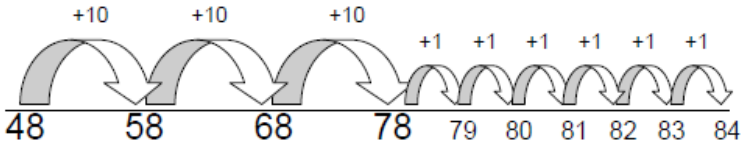
28 + 6 = 34



NB: When initially introducing written strategies, simpler calculations (which would normally be solved mentally) may be used to demonstrate the method before progressing onto more challenging calculations.

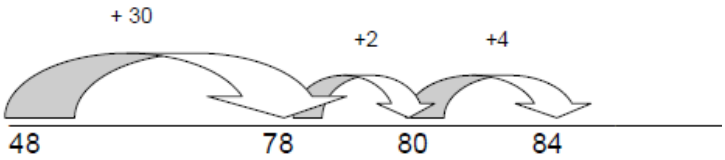
28 + 30 = 58





48 + 36 = 84

Partition into tens and ones when adding.



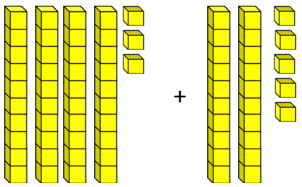
Begin to use more efficient jumps.

Also, use the partitioning method to add two two-digit numbers.

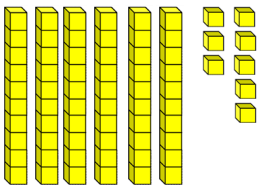
- **Partition the numbers into tens and ones.**
- **Add the ones together and then add the tens together.**
- **Recombine to give the answer.**

43 + 25 = 68

40 3 20 5



40 + 20 = 60
3 + 5 = 8
60 + 8 = 68



- **Then move on to calculations that bridge the tens:**

48 + 36 = 40 + 8 + 30 + 6

40 + 30 = 70
8 + 6 = 14
70 + 14 = 84

48 + 36 = 84

Year 3 Objectives

Add and subtract numbers mentally, including:

- **a three-digit number and ones;**
- **a three-digit number and tens;**
- **a three-digit number and hundreds.**

Add numbers with up to three digits, using formal written method of columnar addition, where appropriate.

A CPA approach towards calculation should continue to be used, ensuring a range of models and images support children's learning.

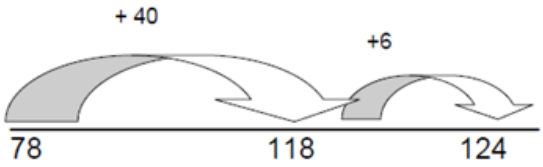
Mental Strategies

- Using place value to partition and count on / add ones, tens and hundreds.
- Use number lines to support visualisations.

Written Strategies

Further develop the use of the empty number line with calculations that bridge 100:

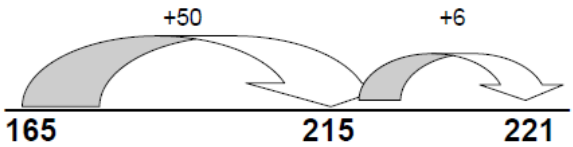
78 + 46 = 124



NB: You may wish to support the addition of the multiple of ten by showing this in jumps of ten to begin with.

… and with addition of a three-digit and a two-digit number:

165 + 56 = 221



Further develop the partitioning method with calculations that bridge 100:

85 + 37 = 80 + 5 + 30 + 7

5 + 7 = 12

80 + 30 = 110

110 + 12 = 122

85 + 37 = 122

The partitioning method can also be used with three-digit numbers.

Expanded methods for addition

63 + 32 = 95

60 + 3

+ 30 + 2

90 + 5 = 95

Partition the numbers into tens and ones/units.

Add the ones together and then add the tens together.

Recombine to give the answer.

6 3

+ 3 2

5 (3 + 2)

9 0 (60 + 30)

9 5

Add the least significant digits (ones) together first and then the tens.

Recombine to give the answer.

Calculations that bridge tens:

68 + 24 = 92

60 + 8

+ 20 + 4

80 +12 = 92

6 8

+ 2 4

1 2 (8 + 4)

8 0 (60 + 20)

9 2

Compact method for addition

6 3

+ 3 2

9 5

Reinforce the language of place value to ensure understanding:

Three add two equals five. Write five in the units column.

Exchanging to carry across columns:

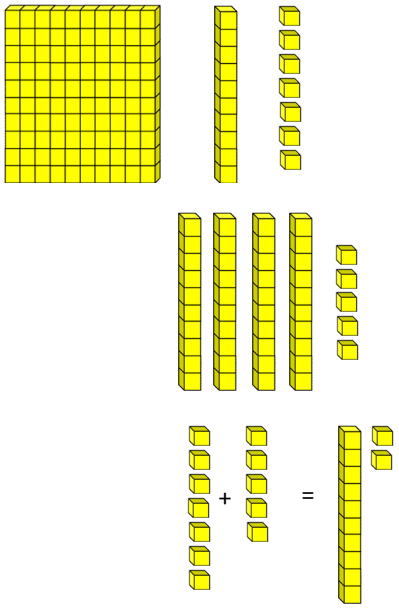
1 1 7

+ 4 5

1 6 2

1

Written methods should be supported with the use of equipment, through a CPA approach. e.g. using Base 10 materials:



Here, the **seven ones** from 117, and **five ones** from 45, combine to make **twelve ones**.

This would then be exchanged into **one ten and two ones**.

When the numbers are re-combined, this leaves **one hundred, six tens and two ones.**

Year 4 Objectives

Add and subtract numbers mentally, including:

- **a four-digit number and ones;**
- **a four-digit number and tens;**
- **a four-digit number and hundreds;**
- **a four-digit number and thousands.**

Add and subtract numbers with up to 4 digits, using formal written method of columnar addition, where appropriate.

A CPA approach towards calculation should continue to be used, ensuring a range of models and images support children's learning.

Mental strategies

- Using place value to partition and count on / add ones, tens and hundreds.
- Use number lines to support visualisations.

Written Strategies

Continue to teach the use of empty number lines with three and four digit numbers, as appropriate (see Year 3 guidance).

Further develop the formal written method of addition with three-digit numbers, revisiting the expanded method first, if necessary.

176 + 147 = 323

```
  1 4 7
+ 1 7 6
  3 2 3
  1 1
```

Use the language of place value to ensure understanding:

'Seven add six equals 13. Write three in the units column and 'carry' one across into the tens column (10).

40 add 70 and the ten that we carried equals 120.

Write 2 in the tens column (20) and 'carry' 1 across into the hundreds column (100).

Year 5 Objectives

Add and subtract numbers mentally, with increasingly large numbers.
Add whole numbers with more than 4 digits, including using formal written method (column addition).

A CPA approach towards calculation should continue to be used, ensuring a range of models and images support children's learning.

Mental strategies

- Using place value to partition and count on, including decimals.
- Using estimating and rounding strategies.

Written Strategies

Use the formal written method for the addition of decimal numbers:

154.75 + 233.82 = 388.57

$$\begin{array}{r} 233.82 \\ +154.75 \\ \hline 388.57 \\ {\scriptstyle 1\ 1} \end{array}$$

Place value counters could be used to support these calculations:

Ensure that the decimal points line up. Continue to use the language of place value to ensure understanding.

Year 6 Objectives

Perform mental calculations, including with mixed operations and large numbers.

No objectives have been included in the programmes of study explicitly related to written methods for addition in Y6. However, there is an expectation that children will continue to practise and use the formal written method for larger numbers and decimals and use these methods when solving problems.

Our aim is that, by the end of Y6, children use mental methods (with jottings) when appropriate, but for other calculations, they use an efficient, formal written method accurately and with confidence.

Subtraction

Models and Images

Number tracks

Bead strings

Number lines [marked and unmarked]

Base 10

Place value counters

Place value (arrow) cards

Ten frames

Numicon

Cuisenaire

Counting sticks

Hundred squares

Bar model

IT resources (e.g. MyMaths, Maths Works, ITPs)

Key Vocabulary

Subtract	Less than
Take away	Fewer than
Take from	Decrease by
Distance between	Deduct
Difference	Reduce
Count back/on	Minus
Inverse	Exchange

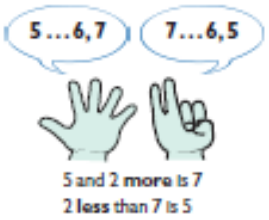
Children need to understand that subtraction is not commutative or associative.

Year 1 Objectives

- **Given a number, identify one less.**
- **Read, write and interpret mathematical statements involving subtraction (-) and the equals (=) sign.**
- **Subtract one-digit and two-digit numbers within 20, including zero.**
- **Solve missing number problems e.g. 7 = ☐ - 9**

Mental Strategies

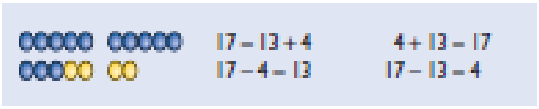
- Children will practise counting back from any number: 'Put seven in your head and count back two.'



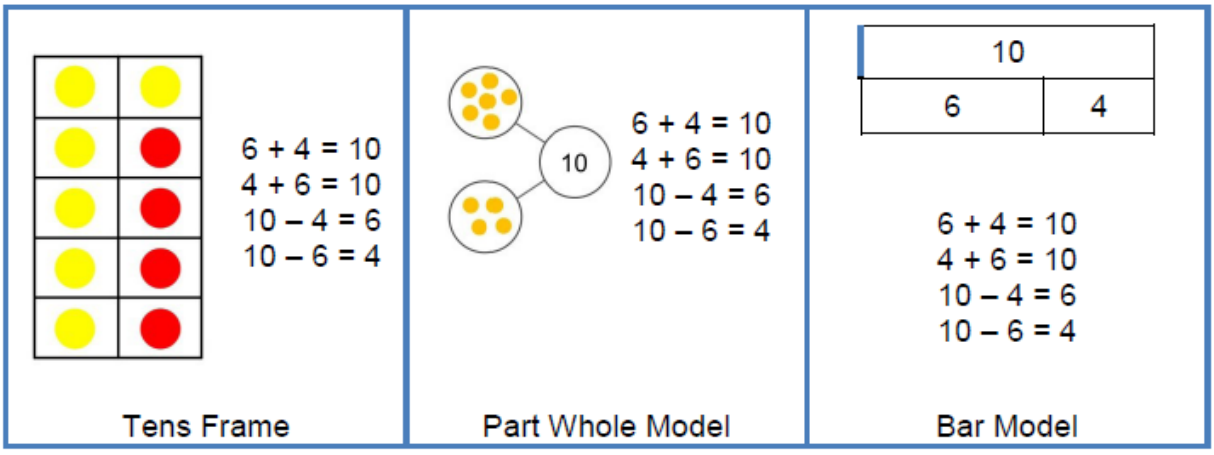
- Initially, use a number track or bead strings to count back for subtraction, counting back from the largest number, progressing to using number lines.

- Use number bond facts to 10 and 20: **10-5=5 7=10-3 15=20-5**
- Use concrete and pictorial representations to solve missing number problems:

17 - ☐ = 4



- Use the inverse of addition to help with subtraction.



Year 2 Objectives

Subtract numbers using concrete objects, pictorial representations, and mentally, including:

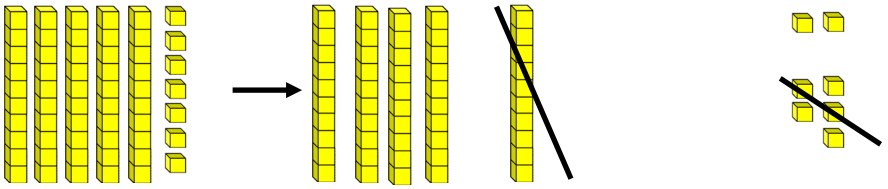
- **a two digit number subtract ones;**
- **a two digit number subtract tens;**
- **two-digit numbers subtract two-digit.**

Mental Strategies

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Use place value and say 1 less and 10 less than any two digit number

- Use place value to partition:: **57-15 is 7-5 = 2 then 50-10 = 40 then 40 + 2 = 42**



- Use place value to partition then adjust:

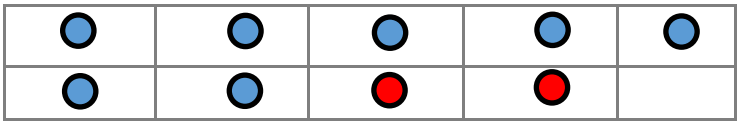
  45 - 9 becomes 45 - 10 = 35 then 35 + 1 = 36

  45 - 21 becomes 45 - 20 = 25 then 25 - 1 = 24

- Count back in multiples of 10: **76 - 20 is 76 - 10 - 10 saying 76, 66, 56**
  e.g. using hundred squares / base ten / number lines

- Use patterns of known facts: **6 - 3 = 3, so 36 - 3 = 33 and 76 - 3 = 73**

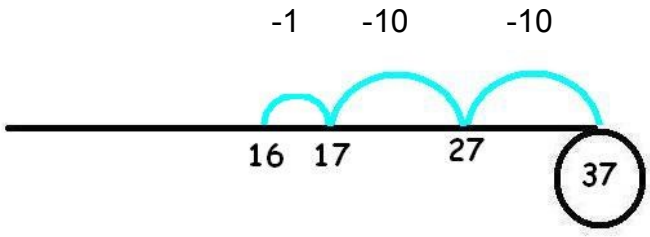
- Use known facts to bridge through 10: **57 - 9 as 57 - 7 - 2 = 48**
  e.g. use tens frames / base ten / number line to represent.



Written Strategies

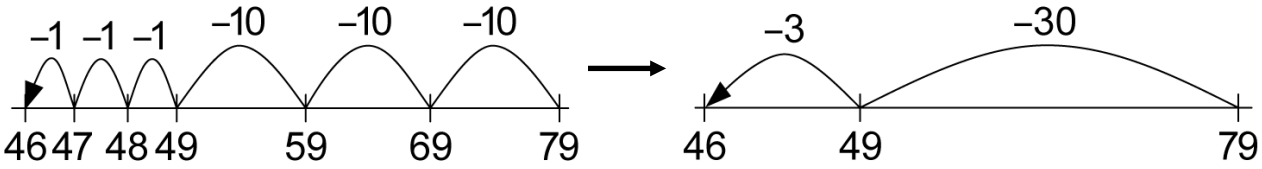
- Count back in ones and tens within 100 (e.g. using an empty number line)

37-21 = 16



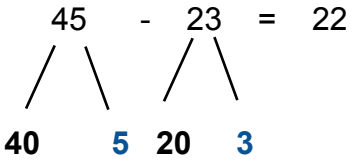
NB: When initially introducing written strategies, simpler calculations (which would normally be solved mentally) may be used to demonstrate the method before progressing onto more challenging calculations.

- Begin to use more efficient jumps: 79-33 = 46



At this stage, children can be introduced to counting forwards to find the difference - if you feel they are ready for it.

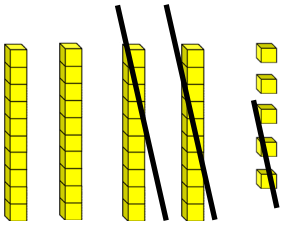
- **Partition the numbers into tens and ones.**
- **Subtract the ones first and then subtract the tens.**
- **Recombine to give the answer.**



40 - **20** = 20

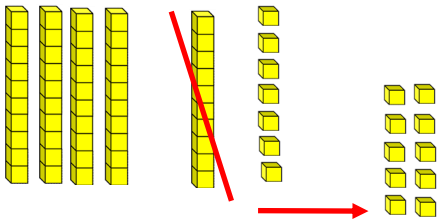
5 - **3** = 2

20 + 2 = 22



- **Then move on to calculations that bridge the tens – Exchange:**

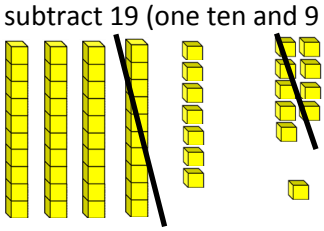
e.g. 57 - 19 = 38



- **Exchange** a ten stick for 10 ones

(This example shows 57 becoming 40 + 17)

- Then subtract 19 (one ten and 9 ones)



This approach helps to consolidate children's understanding of place value and builds the foundations for the use of more formal written methods.

Year 3 Objectives

Subtract numbers mentally, including:

- **a three-digit number subtract ones;**
- **a three-digit number subtract tens;**
- **a three-digit number subtract hundreds.**

Subtract numbers with up to three digits, using formal written method of columnar subtraction, where appropriate.

A CPA approach towards calculation should continue to be used, ensuring a range of models and images support children's learning.

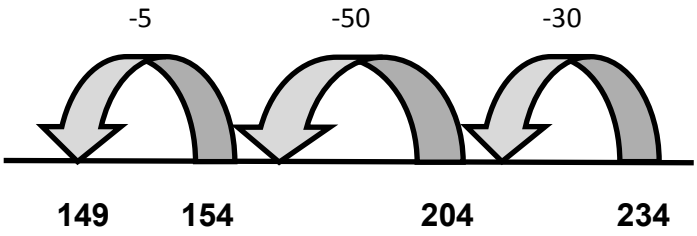
Mental Strategies

- Using place value to partition and subtract ones, tens and hundreds.
- Use number lines to support visualisations.

Written Strategies

- Further develop the use of the empty number line with calculations that bridge 100:

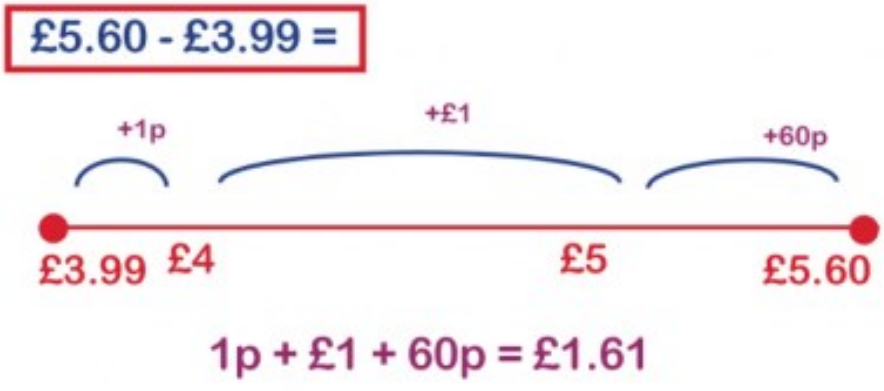
  234- 85 = ☐



NB: You may wish to support the subtraction of the multiple of ten by showing this in jumps of ten to begin with.

- Develop subtraction in a different context: £5.60 - £1.61 = ☐

- Find the difference by counting on: £5.60 - ☐ = £3.99



Expanded methods for subtraction

Subtraction which does not bridge the tens:

84 - 22 = 62

80 + 4

- 20 + 2

60 + 2 = 62

Partition the numbers into tens and ones.

Subtract the ones and then subtract the tens.

Recombine to give the answer.'

8 4

- 2 2

2 (4-2)

60 (80-20)

62

Subtract the ones with-out partitioning, then subtract the tens.

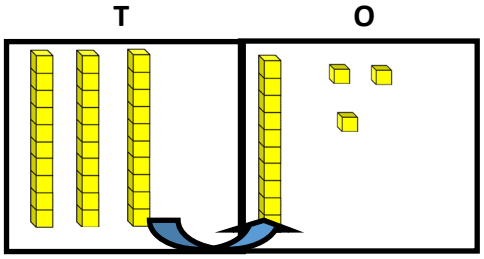
Add the total of the tens and ones.

Calculations that bridge ten (show aside place value counters or dienes if needed)

4 3 → 30 13 (40 crossed out)

- 2 7 → - 20 7

10 + 6 = 16



"Exchange" a ten stick from the tens and place them in the ones, then subtract. Recombine to give the answer.

Compact method for subtraction

6 3

- 3 2

3 1

Reinforce the language of place value to ensure understanding:

Three take away two equals one. Write 1 in the ones column.

Exchanging to carry across columns:

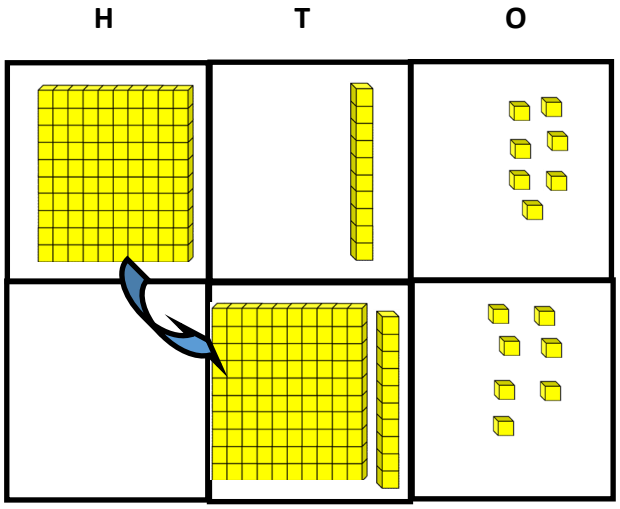
1

1 1 7 (1 crossed out)

- 4 5

7 2

Written methods should be supported with the use of equipment, through a CPA approach. E.g. Using Base 10 materials:



Here, one hundred has been "exchanged" from the hundreds column and moved into the tens, making 110.

110-40 = 70 7-5 = 2

Year 4 Objectives

Subtract numbers mentally, including:

- **a four-digit number subtract ones;**
- **a four-digit number subtract tens;**
- **a four-digit number subtract hundreds;**
- **a four-digit number subtract thousands.**

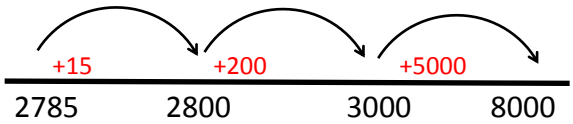
Subtract numbers with up to 4 digits, using formal written method of columnar addition, where appropriate.

A CPA approach towards calculation should continue to be used, ensuring a range of models and images support children's learning.

Mental strategies

- Using place value to partition and count back ones, tens and hundreds.
- Use number lines to support visualisations.

e.g. Find the difference by counting up 8000—2785 = 5215



Written Strategies

Continue to teach the use of empty number lines with three and four digit numbers, as appropriate (see Year 3 guidance).

Further develop the formal written method of subtraction with three-digit numbers, revisiting the expanded method first, if necessary.

Use the language of place value to ensure understanding:

3975

- 122

3853

5 subtract 2 is 3 — write 3 in the ones column.

Year 5 Objectives

- **Subtract numbers mentally, with increasingly large numbers.**
- **Subtract whole numbers with more than 4 digits, including using formal written method (column subtraction).**

A CPA approach towards calculation should continue to be used, ensuring a range of models and images support children's learning.

Mental strategies

- Using place value to partition and count back including decimals.
- Using estimating and rounding strategies.

Written Strategies

Use the formal written method for the subtraction of decimal numbers:

725.75 - 233.82 = 491.93

$$\begin{array}{r} {}^{6}\not{7}\,{}^{1}2\,{}^{4}\not{5} \cdot {}^{1}7\,5 \\ -\ 2\ 3\ 3 \cdot 8\ 2 \\ \hline 4\ 9\ 1 .\ 9\ 3 \end{array}$$

Place value counters could be used to support these calculations:

Ensure that the decimal points line up. Continue to use the language of place value to ensure understanding e.g. 5 hundredths subtract 2 hundredths.

Year 6 Objectives

Perform mental calculations, including with mixed operations and large numbers.

No objectives have been included in the programmes of study explicitly related to written methods for subtraction in Y6. However, there is an expectation that children will continue to practise and use the formal written method for larger numbers and decimals and use these methods when solving problems.

Our aim is that, by the end of Y6, children use mental methods (with jottings) when appropriate, but for other calculations, they use an efficient, formal written method accurately and with confidence.

Multiplication

Models and Images

Number tracks

Bead strings

Number lines [marked and unmarked]

Base 10

Place value counters

Place value (arrow) cards

Ten frames

Numicon

Cuisenaire

Counting sticks

Hundred squares

Bar model

IT resources (e.g. MyMaths, Maths Works, ITPs)

Key Vocabulary

Multiply

Multiplication

Times

Lots of

Groups of

Sets of

Product

Multiple

Double

Factors

Repeated addition

Commutative

Distributive

Associative

Year 1 Objectives

- **Count in multiples of twos, fives and tens (to the 10th multiple).**
- **Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.**

Mental Strategies

- Make Equal Groups:

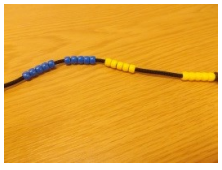
Can you make the cubes into towers of 2?

Put the teddy bears into groups of 3.

- Children count repeated groups of the same size in practical contexts to understand multiplication as repeated addition:

2 groups of 10 =20 10+10=20 Double 10 is 20. 10 groups of 2 =20 2+2+2+2+2+2+2+2+2+2=20	Three groups of five. How many altogether? 5, 10, 15	2+2+2=6 3+3=6 2 rows of 3 = 6 3 rows of 2=6

- Children count in 2s, 5s and 10s, using a variety of concrete resources and contexts.



- Doubling - Use concrete and pictorial representations to calculate doubles to 10:

'double 2 is 4' 2 + 2 = 4 or 4 = 2 + 2

'double 3 is 6' 3 + 3 = 6 or 6 = 3 + 3

Year 2 Objectives

- **Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.**
- **Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs.**
- **Show that the multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.**
- **Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in context**.

Mental Strategies

- **Count in 2s, 3s, 5s, 10s**

Children count in steps, using a variety of concrete resources.

- **Use arrays to support multiplication**

Children are given opportunities to explore, make and describe arrays.

6 + 6 = 12

6 x 2 = 12

2+2+2+2+2+2=12

2 x 6 = 12

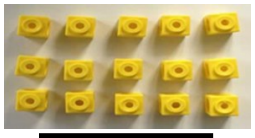
4+4+4+4+4+4=24

4 x 6 = 24

6 + 6 + 6 + 6 = 24

6 x 4 = 24

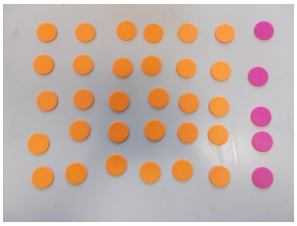
5 x 3 = 15



3 x 5 = 15

Rotate arrays to show that multiplication of two numbers can be done in any order **(commutative law)**

- Use arrays to support with calculating unknown facts, from known facts by partitioning numbers to create equivalent calculations (distributive law): e.g.



7 x 5 = 6 x 5 + 1 x 5

6 x 5 = 30

1 x 5 = 5

7 x 5 = 35

Children will use a range of vocabulary to describe multiplication and use practical resources, pictures, diagrams and the x and = signs to record.

- **Repeated Addition**

Children should be shown the different ways this picture can be recorded using words, numbers and signs.

'There are 3 groups of 10 pens. How many pens altogether?'

'10 + 10 + 10 = 30' '3 groups of 10'

'3 times ten' '3 lots of ten'

'3 x 10 = 30' '10 x 3 = 30'

- Use the bar model to help the children to visualise the concept of multiplication.

30		
10	10	10

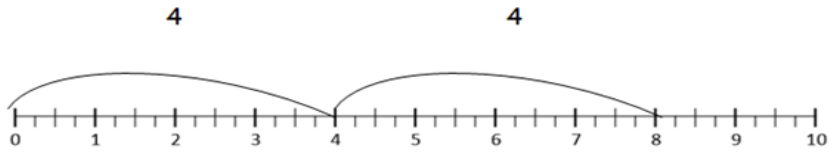
Year 3 Objectives

- **Recall and use multiplication facts for the 3, 4 and 8 multiplication tables (continue to practise the 2, 5 and 10 multiplication tables).**
- **Count in steps of 4, 8, 50 and 100.**
- **Write and calculate mathematical statements for multiplication, using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental strategies and progressing to a formal written method.**
- **Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems, in which n objects are connected to m objects.**

Mental Strategies

- **Count in steps of 4,8, 50 and 100**

Oral counting, supported by images, as appropriate. e.g. colour the multiples on a 1-100 grid; use hops along a landmarked number line.



Make links between 2s, 4s and 8 times tables; 3s and 6 times tables.

Use the knowledge of the commutative law to assist with learning times tables facts.

e.g. if they know 5 x 8 = 40, they know 8 x 5 = 40 too.

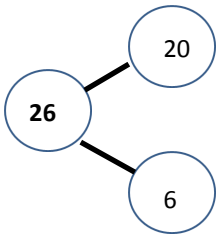
- **Doubling**

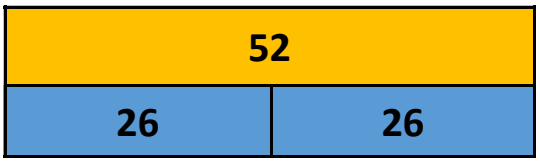
Double numbers to 50 by partitioning into tens and ones, doubling and recombining e.g. to double 26, first partition into 20 and 6, then:

DOUBLE 20 = 40

DOUBLE 6 = 12

40 + 12 = 52





- **Use known facts**

Multiply multiples of 10 by single digit numbers, using known number facts; links to place value

e.g. 3 x 8 = 24 therefore 30 x 8 = 240

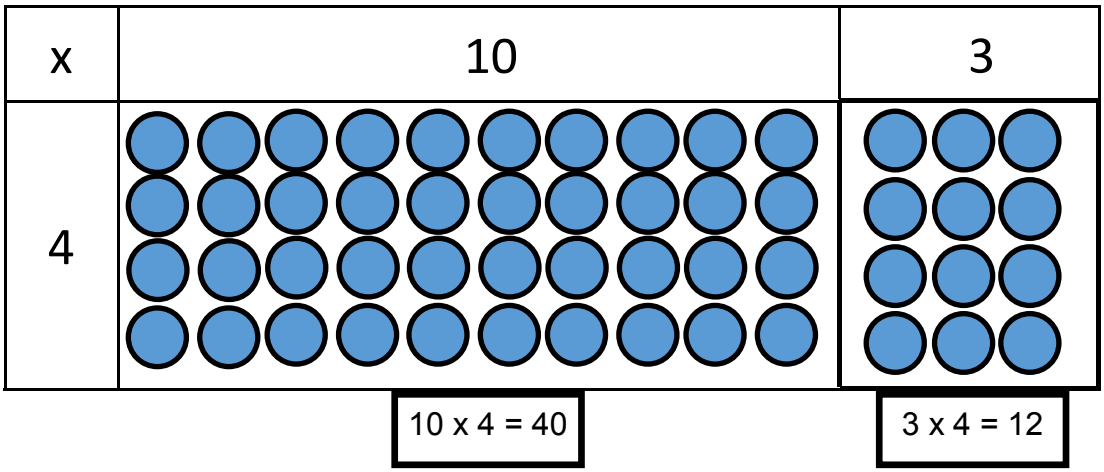
Partition teen numbers to multiply by a single digit number (distributive law)

e.g. 14 x 3 becomes 10 x 3 + 4 x 3

Written Strategies

Grid method - partitioning for multiplication of a teen number by a one-digit number.

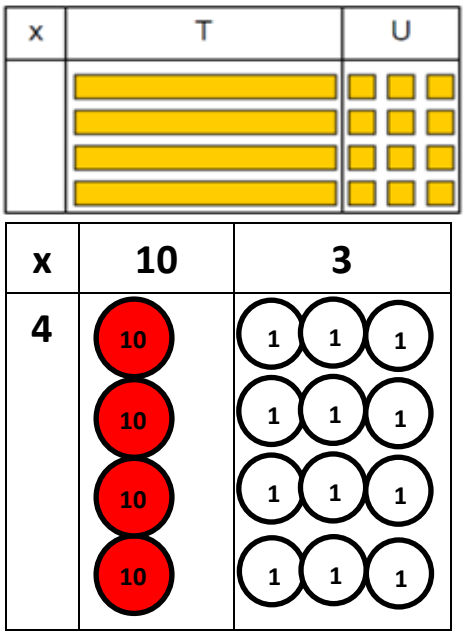
- Use arrays to demonstrate partitioning into the multiples of ten and one:



In this example 13 x 4 is partitioned into

10 x 4 and 3 x 4.

The two parts should be added together to find the total.



- **Counters can be replaced by Base 10 for efficiency.**
- **Make links between using Base 10 materials and place value counters.**
- **Once the concept of partitioning multiples is secure, children can move from using equipment to writing numbers.**

x	10	3
4	40	12

10 x 4 = 40

3 x 4 = 12

40 + 12 = 52

- **Expanded column method:**

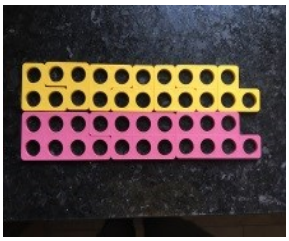
$$\begin{array}{r} 10 + 3 \\ \times \quad 4 \\ \hline 12 \quad (3 \times 4) \\ +\ 40 \quad (10 \times 4) \\ \hline 52 \end{array}$$

Partition 13 into 10 + 3 then multiply each number by 4. Add the partial products (12 and 40) together.

Year 4 Objectives

- **Recall multiplication and division facts for multiplication tables up to 12 × 12.**
- **Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers.**
- **Recognise and use factor pairs and commutativity in mental calculations.**
- **Multiply two-digit and three-digit numbers by a one-digit number, using a formal written layout.**
- **Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems, such as: n objects are connected to m objects.**

Mental Strategies



- **Use concrete resources to illustrate commutativity and factor pairs.**

eg. 3 x 7 = 7 x 3 = 21

- **Count in steps of 6, 7, 9, 25 and 1000.**

Oral counting forwards and backwards, using apparatus, links to prior learning and commutativity as appropriate.

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

- **Use number facts.**

Learn the multiplication facts to 12 x 12:

A multiplication square can be useful for finding commutative facts, spotting easy patterns and identifying tricky facts that need to be learnt.

- **Use place value knowledge.**

Multiply whole numbers and decimals to 1dp by 10, 100 and 1000, using understanding of place value and other known multiplication facts.

Written Methods

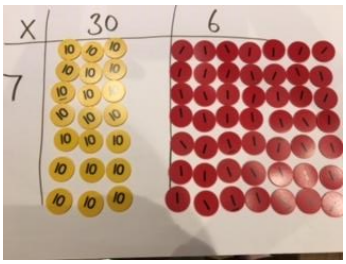
Children should be progressing onto formal written methods for appropriate calculations. However, the use of the grid method along with concrete apparatus to support this, is strongly recommended and is appropriate to enhance a greater depth of understanding.

- **Grid Method**

Further develop the grid method for two-digit numbers multiplied by a one-digit number.

Children can represent the work they have done with equipment, jottings or numbers in a way that they understand.

In this example, the calculation 36 x 7 is shown using concrete resources, as well as numbers:



X	30	6
7	210	42

Remember to add the partial products to find the total 210 + 42 = 252

- **Expanded column method**

36 x 7 =

30 + 6

X 7

4 2 (7x6)

2 1 0 (7x30)

2 5 2

Begin by partitioning the 36 into tens and ones.

Multiply each part by 7, showing the calculation that has been done.

Add together the partial products to get a total.

- **Short multiplication (formal method)**

36 x 7 =

3 6

x 7

2 5 2

4

Multiply 6 by 7 to get 42. Place the 2 in the units column and the 4 (4 tens) underneath the tens column.

Multiply 3 (3 tens) by 7 to get 21. Place these in the remaining columns, remembering to add the 4 tens to the 1.

N.B Continue to reinforce the children's understanding of place value by ensuring they recognise that the 3 is worth 30 and the 21 is worth 210.

Year 5 Objectives

- **Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.**
- **Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.**
- **Establish whether a number up to 100 is prime and recall prime numbers up to 19.**
- **Multiply numbers mentally, drawing upon known facts.**
- **Multiply whole numbers and those involving decimals by 10, 100 and 1000.**
- **Recognise and use square numbers and cube numbers, as well as the notation for squared (^{2}) and cubed (^{3}).**
- **Multiply numbers up to 4 digits by a one- or two-digit number, using a formal written method, including long multiplication for two-digit numbers.**
- **Solve problems involving multiplication, including using their knowledge of factors and multiples, squares and cubes.**
- **Solve problems involving multiplication, including scaling by simple fractions and problems involving simple rates.**

Mental Strategies

Children should continue to use a range of mental strategies to support their work, including use of doubles and halves, number facts and place value knowledge.

- **Near multiples**

Multiply using near multiples by rounding e.g. 32 x 29 becomes (32 x 30) – 32

- **Partitioning**

Use partitioning to multiply two-digit and three-digit numbers by a single-digit number e.g. 402 x 6 becomes 400 x 6 and 2 x 6.

Use partitioning to multiply decimals by a single-digit number e.g. 4.5 x 3 becomes 4 x 3

and 0.5 x 3.

- **Understanding of factors and multiples**

Use knowledge of factors and multiples in multiplication e.g.

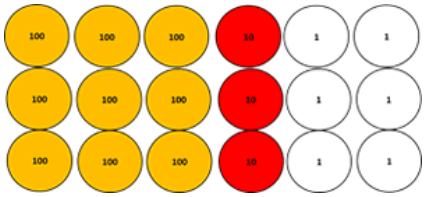
35 x 4 = 7 x 5 x 2 x 2 --> this could be solved as 7 x 2 x 10 or 70 x 2 etc.

Written Methods

Build on the work covered in Y4 with **the formal method of short multiplication** (two-digit number multiplied by a one-digit number).

When children are confident, introduce multiplication by a two-digit number.

If necessary, return to the **grid method and/or expanded method** first:



x	300	10	2
3	900	30	6

Add the partial products 900 + 30 + 6 = 936

3 1 2

x 3

6 (3x2)

3 0 (3x10)

9 0 0 (3x300)

9 3 6

- **Expanded long multiplication (two-digit numbers multiplied by a teen number):**

23 x 13 = 299

23

x 13

9 (3 x 3)

60 (20 x 3)

30 (3 x 10)

200 (20 x 10)

299

X	20	3
10	200	30
3	60	9

Add the partial products, using the most appropriate method.

200 + 60 + 30 + 9 = 299

Use the language of place value as the method is demonstrated:

First, multiply 3 by 3 to give 9.

Next, multiply 20 by 3 to give 60.

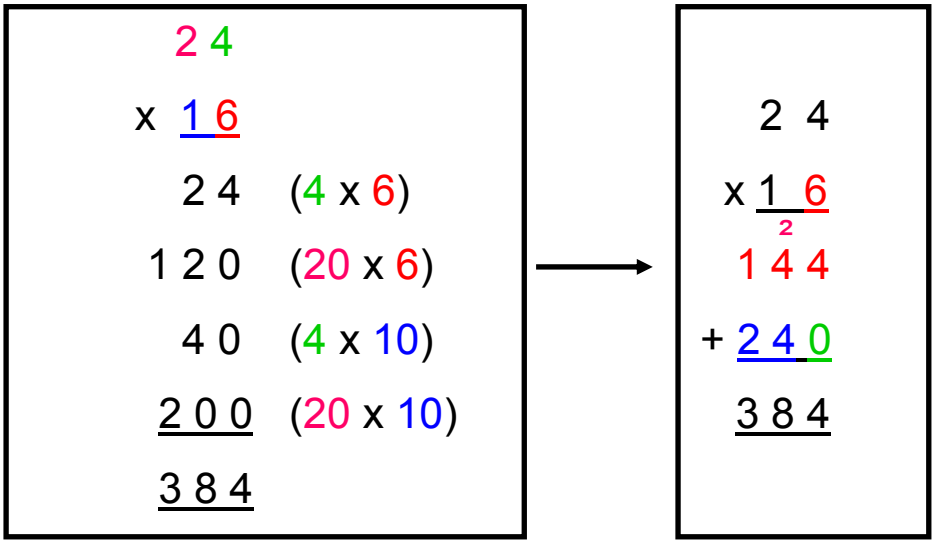
Now, multiply 3 by 10 to give 30.

Finally, multiply 20 by 10 to give 200.

Add the partial products up.

- **Compact long multiplication (formal method)**

24 x 16 = is shown below as an expanded method and then in compact form:



Use the language of place value as the method is demonstrated:

First, multiply 4 by 6 to give 24; record the 4 in the units column and carry the 20 as 2 tens in the tens column.

Next, multiply 20 by 6 to give 120; remember to add the 2 tens carried 120 + 20 = 140. Record 4 tens in the tens column and 1 hundred in the hundreds column.

Next, write a zero in the units column because you are multiplying by 10 (so the product will end in a zero which will be used as a place holder).

Then, multiply 4 by 10 to give 40; record a 4 in the tens column.

Now, multiply 20 by 10 to give 200 which is recorded with a 2 in the hundreds column.

Then add the 2 partial products together.

When children are confident with long multiplication, extend with three-digit numbers multiplied by two-digit numbers.

124 × 26 becomes

Answer: 3224

N.B. In this example, the digits that have been carried into the tens and hundreds column have been written above the calculation, rather than within, as in the first example. Either way is appropriate, as long as a school decides on a consistent approach.

- **Progress with short and long multiplication of decimal numbers (initially in the context of money and measures), returning to an expanded method first, if necessary.**

Year 6 Objectives

- **Perform mental calculations, including with mixed operations and large numbers.**
- **Multiply one-digit numbers, with up to two decimal places, by whole numbers.**
- **Identify common factors, common multiples and prime numbers.**
- **Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.**
- **Solve problems involving addition, subtraction, multiplication and division.**
- **Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.**

Mental Strategies

Children should continue to use a range of mental strategies to support their work, including use of doubles and halves, number facts and place value knowledge.

Written Methods

Continue to practise and develop the **formal short multiplication method** and **formal long multiplication method,** with larger numbers and decimals, throughout Y6. Return to an expanded form of calculation initially, if necessary.

- **Formal written method of long multiplication for decimals**

53.2 x 24

```
    1
  5 3 . 2
x 2 4 . 0
  2 1 2 . 8
1 0 6 4 . 0
1 2 7 6 . 8
```

Use the language of place value as the method is demonstrated.

Provide a wide range of contexts, including the use of scaling, units of measure and large whole numbers, as well as decimals to apply the skills. Also, include problem solving which includes more than one operation.

By the end of Y6, children use mental methods (with jottings) when appropriate, but for calculations that they cannot do mentally, they use an efficient, formal written method accurately and with confidence.

Division

Models and Images

Practical objects (e.g. socks, straws, cubes, hoops, pots)

Bar model

PV counters

Diennes

Counting stick

IT Resources (e.g. MyMaths, Maths Works, ITPs)

Key Vocabulary

Share

Divide

Lots of

Groups of

Sets of

Halving

Equally

Halving

Remainders

Inverse

Quotient

Divisor

Dividend

Repeated subtraction

Year 1 Objectives

- **Solve one-step problems involving division by calculating the answer, using concrete objects, pictorial representations and arrays with the support of the teacher.**
- **Count in multiples of twos, fives and tens (to the 10th multiple).**

Mental Strategies

Children will start with practical sharing, using a variety of resources.

They will share objects into equal groups, in a variety of situations.

They will begin to use the vocabulary associated with division in practical contexts.

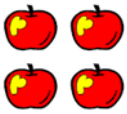
It is important that both concepts of division are introduced and understood, alongside the relevant language. There must be sufficient opportunities to manipulate practical resources, in order to support the learning of the difference between the concepts [grouping and sharing].

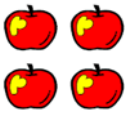
Sharing

"**Share** these 8 apples **equally** between 2 children. How many apples will **each** child have?"

"That' s one for you and one for me; another one for you and another one for me.." etc.

Child 1 Child 2





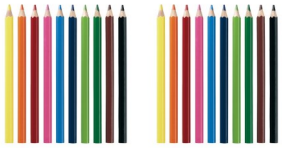
Each child has 4 apples [use of the word 'fair' supports understanding of equal].

"**Share** 20 crayons between 2 pots **equally**. How many crayons in **each** pot?"

Children will move from **sharing** to **grouping** in a practical way.

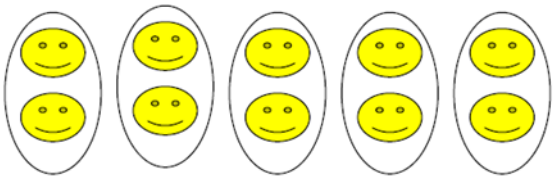
Grouping

"Put 20 crayons into **groups of** 10. How many pots do we need?"

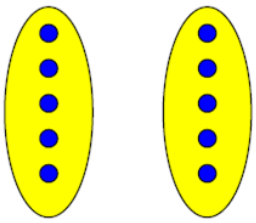


Use practical **arrays** to support early division:

"How many faces altogether? How many groups of 2?"



5 groups of 2



2 groups of 5

Continue to solve problems in practical contexts throughout Y1, developing the language of early division, with appropriate resources.

Halving

Find half of even numbers up to 12, using concrete apparatus. Explore what happens when an odd number is halved.

Year 2 Objectives

- **Recall and use division facts for the 2, 5 and 10 multiplication tables.**
- **Solve problems involving division, using materials, arrays, repeated subtraction, mental methods, and multiplication and division facts, including problems in contexts.**
- **Calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs.**
- **Show that division of one number by another is not commutative [i.e. can be done in any order].**

Mental Strategies

Children will use a range of vocabulary to describe division and use practical resources, pictures, diagrams and the division (÷) and equals (=) signs to record, using multiples that they know.

Halving

Find half of even numbers up to 40. Explore what happens when halving an odd number.

Begin to know the halves of multiples of 10, up to 100 (e.g. half of 70 is 35).

Make links to fractions ½,¼,⅓. Teach explicitly the links between fractions and division (e.g. half is the same as dividing by 2; a third is the same as dividing by 3 and a quarter as dividing by 4).

Number facts

Learn the division facts for 2x, 5x and 10x tables, using relevant vocabulary:

10 divided by 5

10 shared between 5

10 grouped into 5's

Begin to know 5x division facts.

Continue to reinforce concepts of **sharing and grouping** in a practical context, in a variety of ways.

"30 crayons shared equally between 5 pots":

30 divided by 5 = 6 30 ÷ 5 = 6 [sharing]

"30 crayons shared equally between 10 pots":

30 divided by 10 = 3 30 ÷ 10 = 3 [sharing]

"We have 30 crayons and put 10 crayons in each pot. How many pots do we need?"

30 divided by 10 = 3 30 ÷ 10 = 3 [grouping]

"We have 30 crayons and put 3 crayons in each pot. How many pots do we need?"

30 divided by 3 = 10 30 ÷ 3 = 10 [grouping]

Continue to use **arrays** to support division:

How many groups of 3? How many groups of 5?

15 shared between 3 people is....?

15 shared between 5 people is....?

15 divided by 3 = 5

15 divided by 5 = 3

15 ÷ 3 = 5

15 ÷ 5 = 3

Written Division

Count forwards, using a range of strategies (e.g. counting stick):

30 ÷ 5 = 6

We know this because we know that 5 x 6 = 30.

Make the link to repeated subtraction.

Year 3 Objectives

- **Recall and use division facts for the 3, 4 and 8 multiplication tables.**
- **Write and calculate mathematical statements for division, using the multiplication tables that they know, including for 2 digit numbers divided by single-digit numbers, using mental strategies and progressing to formal written methods.**
- **Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems, in which n objects are connected to m objects.**

A CPA approach towards calculation should continue to be used, ensuring a range of models and images support children's learning.

Mental Strategies

Grouping

Recognise that division is not commutative (e.g. 16 ÷ 8 does not equal 8 ÷ 16)

Relate division to multiplication (i.e., they are the inverse of each other):

? x 5 = 30 is the same as 30 ÷ 5 = ?

Divide multiples of 10 by single-digit numbers e.g. 240 ÷ 8 = 30 because we know 24 ÷ 8 = 3

Use number facts

Learn the division facts for 3x, 4x, 5x, 8x, and 10x tables, using a range of division vocabulary

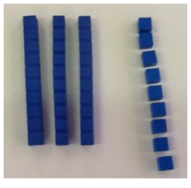
16 divided by 4

16 shared between 4

16 grouped into 4's

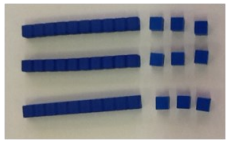
Divide larger numbers mentally by partitioning into multiples of the divisor:

Use arrays to support e.g. 39 ÷ 3 becomes

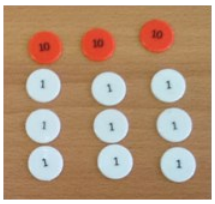


30 ÷ 3 = **10**

10 + 3 = 13

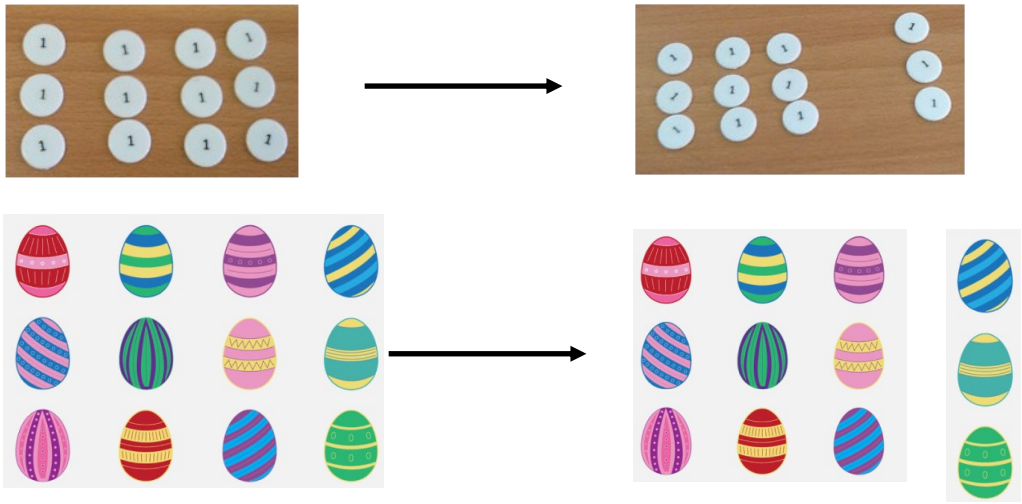


9 ÷ 3 = **3**



Use division facts to find unit and simple non-unit fractions of amounts

e.g. $\frac{3}{4}$ of 12 becomes



12 ÷ 4 = 3

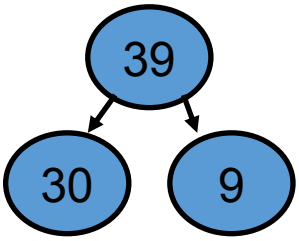
3 x 3 = 9

Written Strategies

Encourage children to use their times tables knowledge:

$$\begin{array}{r|l} & 13 \\ \hline 3 & 39 \end{array}$$

39 ÷ 3



Year 4 Objectives

- **Recall division facts for multiplication tables up to 12 × 12.**
- **Use place value, known and derived facts to divide mentally, including dividing by 1.**
- **Recognise and use factor pairs in mental calculations.**
- **Divide two-digit and three-digit numbers by a one-digit number, using formal written layout.**
- **Solve problems involving division, integer scaling problems and harder correspondence problems, such as, n objects are connected to m objects.**

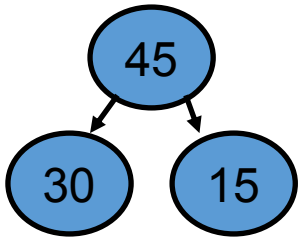
A CPA approach towards calculation should continue to be used, ensuring a range of models and images support children's learning.

Mental Strategies

Grouping

Divide two-digit numbers by a single-digit beyond the children's times table knowledge:

45 ÷ 3 =



Use times table knowledge to support calculations:

10 x 3 = 30

5 x 3 = 15

10 + 5 = 15 [there are 15 groups of 3 in 45]

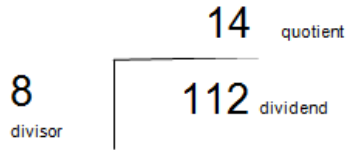
Divide multiples of 100 by single-digit numbers, e.g. 2400 ÷ 8 = 300 because we know 24 ÷ 8 = 3

Written Strategies

To write and calculate mathematical statements for division, using the multiplication tables that the children know:

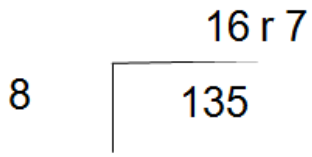
$$32 \div 8 = 4$$

Use the **formal written** layout for division, using numbers beyond the multiplication tables that they know:



"How many eights are there in 112?"

Continue using the **formal written** layout, introducing remainders:



N.B. Remainders are not specifically referred to until Y5 in the National Curriculum.

Division using partitioning (2 digits divided by 1 digit):

$$65 \div 5 = 13$$

First partition 65 into 50 and 15 because we know that 10 x 5 = 50.

Then, divide 50 and 15 by 5 separately.

Next, combine the quotients for 50 and 15 to find the quotient for 65.

$$50 + 15 = 65$$

$$50 \div 5 = 10$$

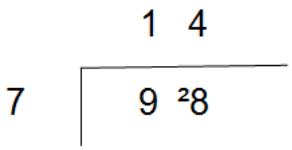
$$15 \div 5 = 3$$

$$10 + 3 = 13$$

N.B. Children will need to be secure in partitioning, in a variety of ways, before this is introduced.

Formal written short division

Use the language of place value throughout and make the link to partitioning.



Year 5 Objectives

- **Divide numbers up to 4 digits by a one-digit number, using the formal written method of short division and interpret remainders appropriately for the context.**
- **Solve problems involving division, including using their knowledge of factors and multiples, squares and cubes.**
- **Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.**
- **Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.**

Mental Strategies

Use the multiples of 10 to support partitioning:

e.g. 186 ÷ 3 becomes 30 x 6 = 180 and 1 x 6 = 6 therefore the quotient is 31

Use number facts

- Use division facts from times tables to divide multiples of powers of 10 of the divisor:

  3600 ÷ 9 = 400 using 36 ÷ 9 = 4
- Use knowledge of multiples and factors, also tests for divisibility, in mental division e.g.

  246 ÷ 6 = 123 ÷ 3
- We know that 525 is divisible by 5 because it ends in a 5 and all multiples of 5 end in either a 5 or a 0
- 525 is divisible by 3 because the sum of its digits is divisible by 3 (5 + 2 + 5 = 12)
- Divide whole numbers by 10,100 and 1000 to give whole number answers, or answers with up to 3 decimal places.

Written Strategies

Continue to practise the formal written method of short division with whole number answers:

184 ÷ 8 = 23 →

```
   2 3
8 | 1 8 ²4
```

Continue to use the language of place value to ensure understanding.

Introduce remainders e.g. 432 ÷ 5 = 86 r 2

```
   8 6 r 2
5 | 4 ³3 2
```

How many 5s are there in 400? Use 5 times table knowledge that 5 x 8 = 40, therefore 5 x 80 = 400. Record with an 8 in the tens column.

Next, how many 5s in 30? Again, use times tables to help 5 x 6 = 30. Record with a 6 in the units column.

Finally, how many 5s in 2? This doesn't work, so the 2 is recorded as a remainder.

The remainder can also be expressed as a fraction: $\frac{2}{5}$ which is the remainder divided by the divisor.

Year 6 Objectives

- **Perform mental calculations, including with mixed operations and large numbers.**
- **Divide numbers up to 4 digits by a two-digit whole number, using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.**
- **Divide numbers up to 4 digits by a two-digit number, using the formal written method of short division and where appropriate, interpreting remainders according to the context.**
- **Solve problems involving addition, subtraction, multiplication and division.**
- **Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.**
- **Use written division methods in cases where the answer has up to two decimal places.**

Mental Strategies

Grouping

Use multiples of 10 of the divisor to divide large numbers:

e.g. 378 ÷ 9 becomes 40 x 9 = 360 and 2 x 9 = 18 therefore the quotient is 42

Use number facts

Use division facts from times tables to divide decimal numbers by single-digit numbers:

2.4 ÷ 6 = 0.4 using 24 ÷ 6 = 4

Identify and use common factors, common multiples and prime numbers in mental division:

288 ÷ 24 = 144 ÷ 12

Use tests for divisibility to aid mental calculation:

Divide whole numbers by 10,100, 1000 and 10,000 to give whole number answers, or answers with up to 3 decimal places.

Know and use equivalence between simple fractions, decimals and percentages including in different contexts.

Recognise a given ratio and reduce a given ratio to its simplest form.

Written Strategies

Continue to practise the formal method of short division, with and without remainders, using the language of place value to ensure understanding (see Y5 guidance).

Formal method of long division

When the divisor is > 12, short division is not appropriate and long division must be used, which involves the repeated subtraction of multiples of the divisor.

How many 15s are there in 400? I know that 15 x 10 = 150 so 15 x 20 = 300. So I will subtract 300 from 432 and record the 20 as 2 tens by placing a 2 in the tens column.

Next, I will subtract 300 from 432, which leaves 132. I can't use 15 x 10 because it is bigger than 132. I know I need to multiply 15 by an even number to give me a multiple of 10, so I will try 15 x 8 = 120, which is lower than 132 so I can use it. To record it, I put 8 in the units column and subtract 120 from 132, which leaves 12.

12 is smaller than 15, so there are no more groups of 15 therefore, 12 is the remainder.

432 ÷ 15 becomes

			2	8 r 12
1	5	4	3	2
		3	0	0
		1	3	2
		1	2	0
			1	2

Answer: 28 remainder 12

Remember the remainder can be expressed as a fraction, which should be reduced to its lowest form, or as a decimal.

432 ÷ 15 becomes

			2	8
1	5	4	3	2
		3	0	0
		1	3	2
		1	2	0
			1	2

15×20

15×8

$\frac{12}{15} = \frac{4}{5}$

The r12 is converted to a decimal, using the knowledge that 1/5 = 0.2 therefore 0.2 x 4 = 0.8 = $\frac{4}{5}$

When children are confident, they can learn this alternative method of recording long division, which expresses the remainder as a decimal, without the need to convert from a fraction.

432 ÷ 15 becomes

			2	8 ·	8
1	5	4	3	2 ·	0
		3	0	↓	
		1	3	2	
		1	2	0	↓
			1	2	0
			1	2	0
					0

Answer: 28·8

Children will need to select the most effective method for each calculation/problem they meet. For example, division involving money will use decimals.